Reinforcement Learning

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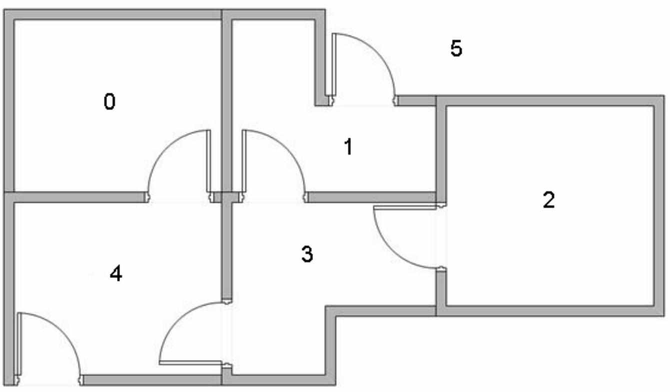
University of Colorado Boulder
LECTURE 22

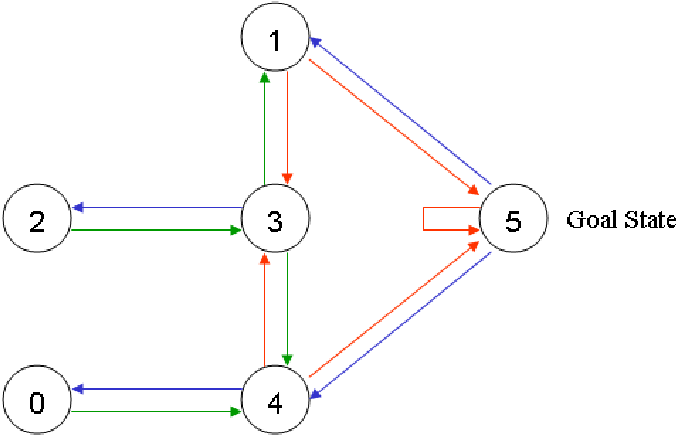
Slides adapted from John McCullock

Content Questions

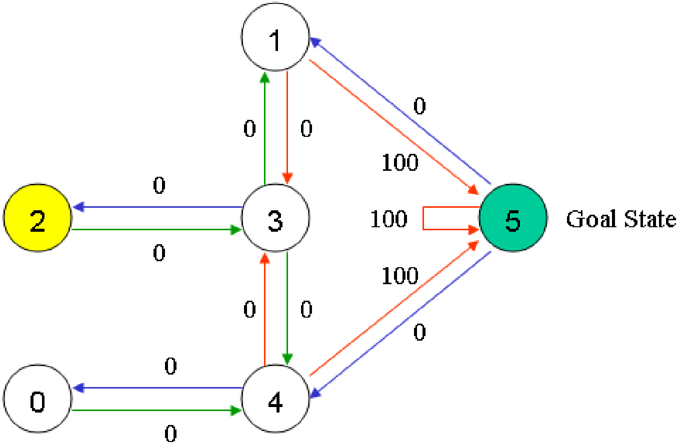
Scenario



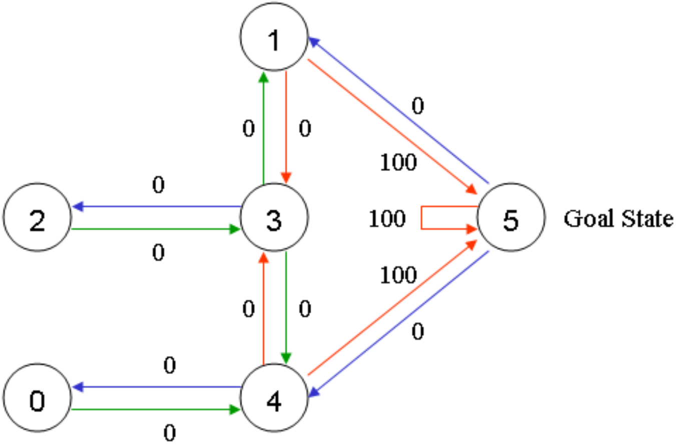
States



Scenario: Escape!



Rewards



Reward Matrix

$$R = \begin{array}{cc} & \begin{array}{cccccc} & & \textbf{Action} & & & \end{array} \\ \textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \left[\begin{array}{cccccc} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{array}\right] \end{array}$$

100	Goal
0	Valid Transition
-1	Impossible

Q-Learning Algorithm

For each s, a initialize table entry $\hat{Q}(s, a) \leftarrow 0$
Observe current state s
Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s, a)$ as follows:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

- $s \leftarrow s'$

Initial Q Matrix

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

- Suppose we start in Room 1
- And we'll go to Room 5 afterward

In Room 5

$$R = \begin{array}{c c}
 & \text{Action} \\
\text{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix}
\end{array}$$

What is the updated Q matrix? ($\gamma = .8$)

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

Updated Q for Room $1 \rightarrow 5$

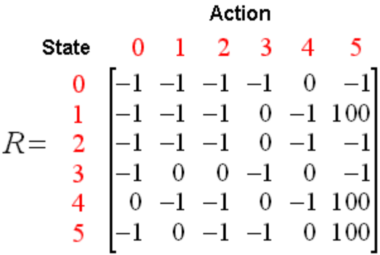
$$\hat{Q}(1,5) = R(1,5) + \gamma \max\left[\hat{Q}(5,0), \ldots \hat{Q}(5,5)\right] \tag{1}$$

Updated Q for Room $1 \to 5$

$$
\begin{align}
\hat{Q}(1,5) &= R(1,5) + \gamma \max \left[\hat{Q}(5,0), \ldots \hat{Q}(5,5) \right] \tag{1} \\
\hat{Q}(1,5) &= 100 + \gamma \cdot 0 \tag{2}
\end{align}
$$

Update Q for Room $5 \rightarrow 1$

$$\hat{Q} = \begin{array}{c c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{3}$$



$$R= \begin{array}{c c} \text{State} & \begin{array}{cccccc} & & \text{Action} & & & \\ 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

Update Q for Room $5 \rightarrow 1$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & \textbf{Action} \\ \textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \end{array}$$

$$R = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix}$$

$$\hat{Q}(5,1) = R(5,1) + \gamma \max \left[\hat{Q}(1,0), \ldots \hat{Q}(1,5) \right] \tag{3}$$

Update Q for Room $5 \rightarrow 1$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

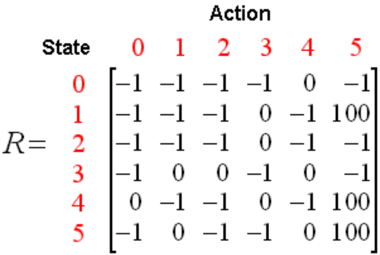
$$R = \begin{array}{c|c} & \textbf{Action} \\ \textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

$$\hat{Q}(5,1) = R(5,1) + \gamma \max \left[\hat{Q}(1,0), \ldots \hat{Q}(1,5) \right] \tag{3}$$

$$\hat{Q}(5,1) = 0 + \gamma \cdot 100 \tag{4}$$

Update Q for Room $1 \rightarrow 3$

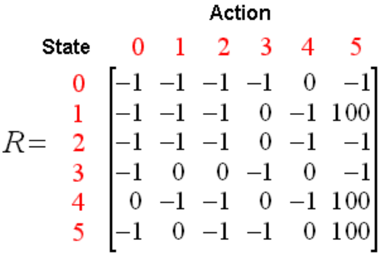
$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \color{red}{80} & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{5}$$



$$R = \begin{array}{c|cccccc} \text{State} & \text{Action} \\ & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

Update Q for Room $1 \rightarrow 3$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



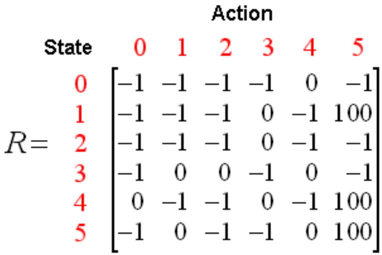
Action

$$R = \begin{array}{c|cccccc} \textbf{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

$$\hat{Q}(1,3) = R(1,3) + \gamma \max \left[\hat{Q}(3,0), \ldots \hat{Q}(3,5) \right] \tag{5}$$

Update Q for Room $1 \rightarrow 3$

$$
\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 80 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$



Action

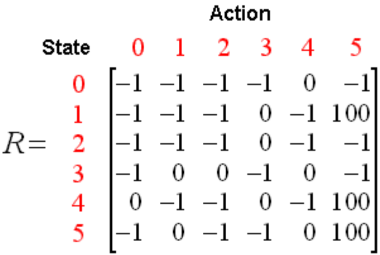
$$
R = \begin{array}{c|cccccc}
\text{State} & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & -1 & -1 & -1 & -1 & 0 & -1 \\
1 & -1 & -1 & -1 & 0 & -1 & 100 \\
2 & -1 & -1 & -1 & 0 & -1 & -1 \\
3 & -1 & 0 & 0 & -1 & 0 & -1 \\
4 & 0 & -1 & -1 & 0 & -1 & 100 \\
5 & -1 & 0 & -1 & -1 & 0 & 100
\end{array}
$$

$$\hat{Q}(1,3) = R(1,3) + \gamma \max \left[\hat{Q}(3,0), \dots \hat{Q}(3,5) \right] \tag{5}$$

$$\hat{Q}(1,3) = 0 + \gamma \cdot 0 \tag{6}$$

Update Q for Room $3 \rightarrow 4$

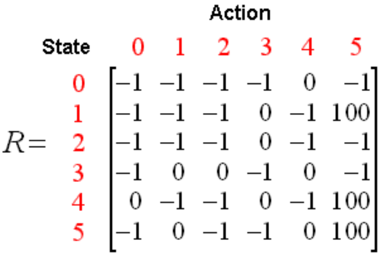
$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \color{red}{0} & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



$$R = \begin{array}{c|cccccc} \textbf{State} & \multicolumn{6}{c}{\textbf{Action}} \\ & 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array} \tag{7}$$

Update Q for Room $3 \rightarrow 4$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



$$R = \begin{array}{c|cccccc} \textbf{State} & \multicolumn{6}{c}{\textbf{Action}} \\ & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

$$\hat{Q}(3,4) = R(3,4) + \gamma \max \left[\hat{Q}(4,0), \ldots \hat{Q}(4,5) \right] \quad (7)$$

Update Q for Room $3 \rightarrow 4$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

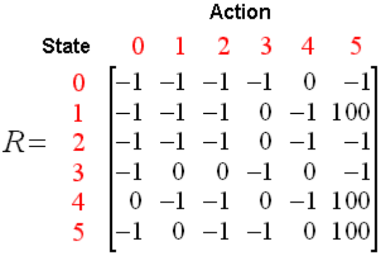
$$R = \begin{array}{c} \textbf{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \textbf{Action} \\ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

$$\hat{Q}(3,4) = R(3,4) + \gamma \max \left[\hat{Q}(4,0), \ldots \hat{Q}(4,5) \right] \tag{7}$$

$$\hat{Q}(3,4) = 0 + \gamma \cdot 0 \tag{8}$$

Update Q for Room $4 \rightarrow 5$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



$$R = \begin{array}{c|cccccc} & & & \text{Action} & & & \\ \text{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array} \quad (9)$$

Update Q for Room $4 \rightarrow 5$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

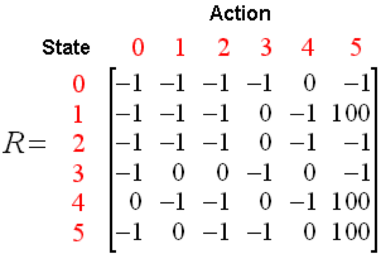
Action

$$R= \begin{array}{c|cccccc} \textbf{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

$$\hat{Q}(4,5) = R(4,5) + \gamma \max \left[\hat{Q}(5,0), \ldots \hat{Q}(5,5) \right] \tag{9}$$

Update Q for Room $4 \rightarrow 5$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



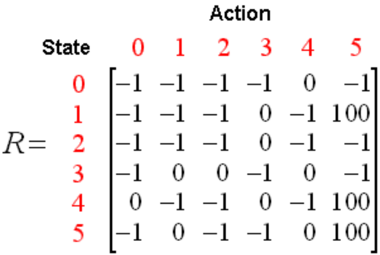
$$R = \begin{array}{c|cccccc} \text{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

$$\hat{Q}(4,5) = R(4,5) + \gamma \max \left[\hat{Q}(5,0), \ldots \hat{Q}(5,5) \right] \tag{9}$$

$$\hat{Q}(4,5) = 100 + \gamma \cdot 80 \tag{10}$$

Update Q for Room $5 \rightarrow 4$

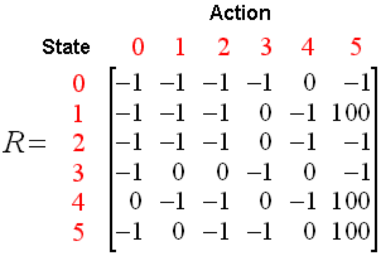
$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 164 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



$$R = \begin{array}{c|c} \text{State} & \text{Action} \\ & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array} \tag{11}$$

Update Q for Room $5 \rightarrow 4$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 164 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



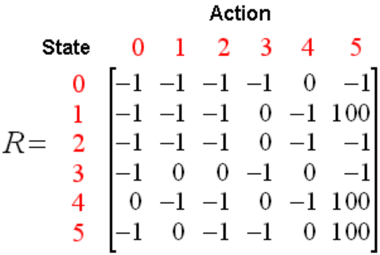
Action

$$R = \begin{array}{c|cccccc} \textbf{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

$$\hat{Q}(5,4) = R(5,4) + \gamma \max \left[\hat{Q}(4,0), \ldots \hat{Q}(4,5) \right] \tag{11}$$

Update Q for Room $5 \rightarrow 4$

$$\hat{Q} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 164 \\ 0 & 80 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



$$R = \begin{array}{c|cccccc} \text{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

$$\hat{Q}(5,4) = R(5,4) + \gamma \max \left[\hat{Q}(4,0), \dots \hat{Q}(4,5) \right] \tag{11}$$

$$\hat{Q}(5,4) = 0 + \gamma \cdot 164 = 131 \tag{12}$$

If you keep going . . .

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{rrrrrr} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{rrrrrr} 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 320 & 0 & 500 \\ 0 & 0 & 0 & 320 & 0 & 0 \\ 0 & 400 & 256 & 0 & 400 & 0 \\ 320 & 0 & 0 & 320 & 0 & 500 \\ 0 & 400 & 0 & 0 & 400 & 500 \end{array}\right] \end{array}$$

If you keep going . . .