Bayesian Nonparametrics and DPMM

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LECTURE 17

Content Questions

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Administrivia

- Feeback on projects
- Work on first deliverable

DPMM

- Don't know how many clusters there are
- Gibbs sampling: change the assignment of one cluster conditioned on all other clusters
- Convergence harder to detect
- Equation

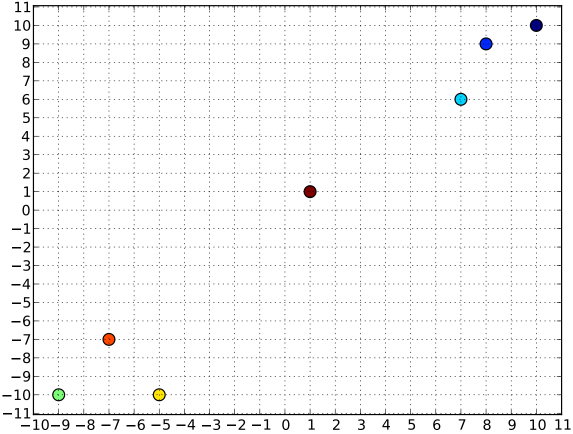
$$p(z_i = k \mid \vec{z}_{-i}, \vec{x}, \{\theta_k\}, \alpha) \propto \begin{cases} \left(\frac{n_k}{n_\cdot + \alpha}\right) \mathcal{N}\left(x, \frac{n\bar{x}}{n+1}, 1\right) & \text{existing} \\ \frac{\alpha}{n_\cdot + \alpha} \mathcal{N}\left(x, 0, 1\right) & \text{new} \end{cases} \tag{1}$$

Simplification

We'll assume that:

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (2)$$

Data



11
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10 11

Sampling point 0

Compute the (proportional) probability of assigning data 0 to a new cluster and cluster 1.

Recall that $\alpha = 0.25$ and

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (3)$$

i	x_1	x_2	z_i
0	10	10	
1	8	9	1
2	7	6	2
3	-9	-10	3
4	-5	-7	4
5	-7	-6	5
6	1	1	6

Sampling point 0

- There are currently 6 clusters

(3)

Sampling point 0

- There are currently 6 clusters

$$p(z_0 = \text{new} \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00000 \tag{3}$$

$$\tag{4}$$

Sampling point 0

- There are currently 6 clusters

$$p(z_0 = \text{new} \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00000 \quad (3)$$

$$p(z_0 = 1 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 4.00 \\ 4.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00029 \quad (4)$$

$$(5)$$

Sampling point 0

- There are currently 6 clusters

$$p(z_0 = \text{new} \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00000 \quad (3)$$

$$p(z_0 = 1 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 4.00 \\ 4.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00029 \quad (4)$$

$$p(z_0 = 2 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 3.50 \\ 3.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00007 \quad (5)$$

$$p(z_0 = 3 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} -4.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (6)$$

$$p(z_0 = 4 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (7)$$

$$p(z_0 = 5 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (8)$$

$$p(z_0 = 6 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (9)$$

Sampling point 0

- There are currently 6 clusters

$$p(z_0 = \text{new} \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}0.00\\0.00\end{array}, \mathbb{1}\right) = 0.04 \times 0.00000 \quad (3)$$

$$p(z_0 = 1 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}4.00\\4.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00029 \quad (4)$$

$$p(z_0 = 2 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}3.50\\3.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00007 \quad (5)$$

$$p(z_0 = 3 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}-4.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \quad (6)$$

$$p(z_0 = 4 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}-2.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \quad (7)$$

$$p(z_0 = 5 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}-3.50\\-3.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \quad (8)$$

$$p(z_0 = 6 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}10.00\\10.00\end{array} \mid \begin{array}{c}0.50\\0.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \quad (9)$$

Sampling point 0

- There are currently 6 clusters

$$p(z_0 = \text{new} \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00000 \tag{3}$$

$$p(z_0 = 1 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} 4.00 \\ 4.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00029 \tag{4}$$

$$p(z_0 = 2 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} 3.50 \\ 3.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00007 \tag{5}$$

$$p(z_0 = 3 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{6}$$

$$p(z_0 = 4 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} -2.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{7}$$

$$p(z_0 = 5 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} -3.50 \\ -3.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{8}$$

$$p(z_0 = 6 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 10.00 \\ 10.00 \end{array} \mid \begin{array}{c} 0.50 \\ 0.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{9}$$

- After normalization:{new: 0.00 1: 0.80 2: 0.19 3: 0.00 4: 0.00 5: 0.00 6: 0.00}

Sampling point 0

- There are currently 6 clusters

$$p(z_0 = \text{new} \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00000 \quad (3)$$

$$p(z_0 = 1 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 4.00 \\ 4.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00029 \quad (4)$$

$$p(z_0 = 2 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 3.50 \\ 3.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00007 \quad (5)$$

$$p(z_0 = 3 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} -4.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (6)$$

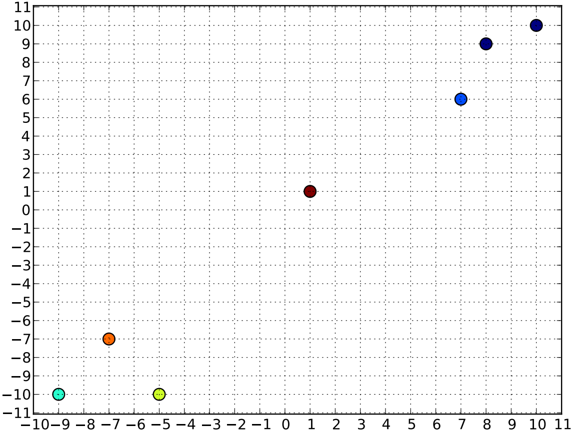
$$p(z_0 = 4 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (7)$$

$$p(z_0 = 5 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (8)$$

$$p(z_0 = 6 \mid \vec{z_{-0}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 10.00 \\ 10.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (9)$$

- After normalization:{new: 0.00 1: 0.80 2: 0.19 3: 0.00 4: 0.00 5: 0.00 6: 0.00}
- New assignment = 1

Assignments after sampling point 0



Sampling point 1

Compute the (proportional) probability of assigning data 1 to clusters 1 and 2.

Recall that $\alpha = 0.25$ and

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (10)$$

i	x_1	x_2	z_i
0	10	10	1
1	8	9	
2	7	6	2
3	-9	-10	3
4	-5	-7	4
5	-7	-6	5
6	1	1	6

Sampling point 1

- There are currently 6 clusters

$$p(z_1 = 1 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 5.00 \\ 5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00674 \tag{10}$$

$$\tag{11}$$

Sampling point 1

- There are currently 6 clusters

$$p(z_1 = 1 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 5.00 \\ 5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00674 \quad (10)$$

$$p(z_1 = 2 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 3.50 \\ 3.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00055 \quad (11)$$

$$(12)$$

Sampling point 1

- There are currently 6 clusters

$$p(z_1 = \text{new} \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00001 \quad (10)$$

$$p(z_1 = 1 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 5.00 \\ 5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00674 \quad (11)$$

$$p(z_1 = 2 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 3.50 \\ 3.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00055 \quad (12)$$

$$p(z_1 = 3 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} -4.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (13)$$

$$p(z_1 = 4 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (14)$$

$$p(z_1 = 5 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (15)$$

$$p(z_1 = 6 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{matrix} 8.00 \\ 9.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00001 \quad (16)$$

Sampling point 1

- There are currently 6 clusters

$$p(z_1 = \text{new} \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}0.00\\0.00\end{array}, \mathbb{1}\right) = 0.04 \times 0.00001 \tag{10}$$

$$p(z_1 = 1 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}5.00\\5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00674 \tag{11}$$

$$p(z_1 = 2 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}3.50\\3.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00055 \tag{12}$$

$$p(z_1 = 3 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}-4.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \tag{13}$$

$$p(z_1 = 4 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}-2.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \tag{14}$$

$$p(z_1 = 5 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}-3.50\\-3.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \tag{15}$$

$$p(z_1 = 6 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}8.00\\9.00\end{array} \mid \begin{array}{c}0.50\\0.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00001 \tag{16}$$

- After normalization:{new: 0.00 1: 0.92 2: 0.08 3: 0.00 4: 0.00 5: 0.00 6: 0.00}

Sampling point 1

- There are currently 6 clusters

$$p(z_1 = \text{new} \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00001 \quad (10)$$

$$p(z_1 = 1 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} 5.00 \\ 5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00674 \quad (11)$$

$$p(z_1 = 2 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} 3.50 \\ 3.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00055 \quad (12)$$

$$p(z_1 = 3 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (13)$$

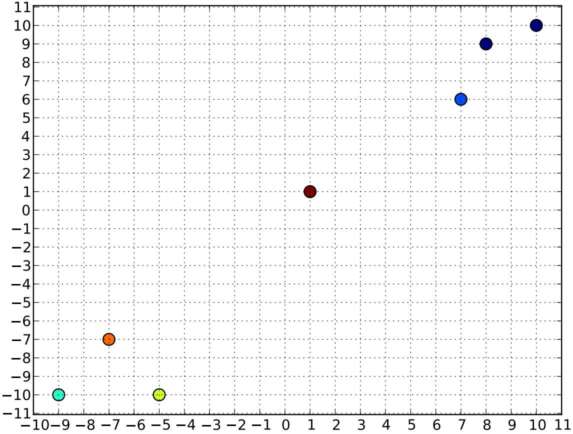
$$p(z_1 = 4 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} -2.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (14)$$

$$p(z_1 = 5 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} -3.50 \\ -3.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (15)$$

$$p(z_1 = 6 \mid \vec{z_{-1}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} 8.00 \\ 9.00 \end{array} \mid \begin{array}{c} 0.50 \\ 0.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00001 \quad (16)$$

- After normalization: $\{$new: 0.00 1: 0.92 2: 0.08 3: 0.00 4: 0.00 5: 0.00 6: 0.00$\}$
- New assignment $= 1$

Assignments after sampling point 1



Sampling point 2

Compute the (proportional) probability of assigning data 2 to cluster 1 (but nothing else; there won't be other options).
Recall that $\alpha = 0.25$ and

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (17)$$

i	x_1	x_2	z_i
0	10	10	1
1	8	9	1
2	7	6	
3	-9	-10	3
4	-5	-7	4
5	-7	-6	5
6	1	1	6

Sampling point 2

- There are currently 5 clusters

$$p(z_2 = 1 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{2.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} 6.00 \\ 6.33 \end{matrix}, \mathbb{1} \right) = 0.32 \times 0.34851 \tag{17}$$

$$\tag{18}$$

Sampling point 2

- There are currently 5 clusters

$$p(z_2 = \text{new} \mid \vec{z}_{-2}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00010 \quad (17)$$

$$p(z_2 = 1 \mid \vec{z}_{-2}, \vec{x}, \alpha) \propto \frac{2.00}{6+0.25}\mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} 6.00 \\ 6.33 \end{matrix}, \mathbb{1} \right) = 0.32 \times 0.34851 \quad (18)$$

$$p(z_2 = 3 \mid \vec{z}_{-2}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} -4.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (19)$$

$$p(z_2 = 4 \mid \vec{z}_{-2}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (20)$$

$$p(z_2 = 5 \mid \vec{z}_{-2}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (21)$$

$$p(z_2 = 6 \mid \vec{z}_{-2}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} 7.00 \\ 6.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00020 \quad (22)$$

Sampling point 2

- There are currently 5 clusters

$$p(z_2 = \text{new} \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{array}{c}7.00\\6.00\end{array} \mid \begin{array}{c}0.00\\0.00\end{array}, \mathbb{1}\right) = 0.04 \times 0.00010 \tag{17}$$

$$p(z_2 = 1 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{2.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}7.00\\6.00\end{array} \mid \begin{array}{c}6.00\\6.33\end{array}, \mathbb{1}\right) = 0.32 \times 0.34851 \tag{18}$$

$$p(z_2 = 3 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}7.00\\6.00\end{array} \mid \begin{array}{c}-4.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \tag{19}$$

$$p(z_2 = 4 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}7.00\\6.00\end{array} \mid \begin{array}{c}-2.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \tag{20}$$

$$p(z_2 = 5 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}7.00\\6.00\end{array} \mid \begin{array}{c}-3.50\\-3.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00000 \tag{21}$$

$$p(z_2 = 6 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}7.00\\6.00\end{array} \mid \begin{array}{c}0.50\\0.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00020 \tag{22}$$

- After normalization:{new: 0.00 1: 1.00 3: 0.00 4: 0.00 5: 0.00 6: 0.00}

Sampling point 2

- There are currently 5 clusters

$$p(z_2 = \text{new} \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 7.00 \\ 6.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00010 \tag{17}$$

$$p(z_2 = 1 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{2.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 7.00 \\ 6.00 \end{array} \mid \begin{array}{c} 6.00 \\ 6.33 \end{array}, \mathbb{1} \right) = 0.32 \times 0.34851 \tag{18}$$

$$p(z_2 = 3 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 7.00 \\ 6.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{19}$$

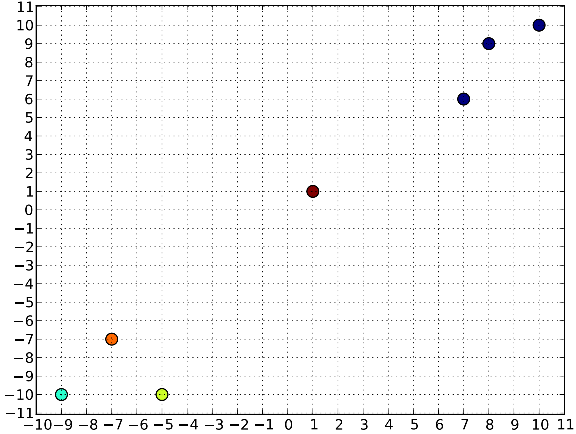
$$p(z_2 = 4 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 7.00 \\ 6.00 \end{array} \mid \begin{array}{c} -2.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{20}$$

$$p(z_2 = 5 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 7.00 \\ 6.00 \end{array} \mid \begin{array}{c} -3.50 \\ -3.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{21}$$

$$p(z_2 = 6 \mid \vec{z_{-2}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 7.00 \\ 6.00 \end{array} \mid \begin{array}{c} 0.50 \\ 0.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00020 \tag{22}$$

- After normalization:{new: 0.00 1: 1.00 3: 0.00 4: 0.00 5: 0.00 6: 0.00}
- New assignment = 1

Assignments after sampling point 2



Sampling point 3

Compute the (proportional) probability of assigning data 3 to cluster 4 and 5.
Recall that $\alpha = 0.25$ and

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (23)$$

i	x_1	x_2	z_i
0	10	10	1
1	8	9	1
2	7	6	1
3	-9	-10	
4	-5	-7	4
5	-7	-6	5
6	1	1	6

Sampling point 3

- There are currently 4 clusters

$$ \tag{23} $$

Sampling point 3

- There are currently 4 clusters

$$p(z_3 = 4 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} -9.00 \\ -10.00 \end{array} \mid \begin{array}{c} -2.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00027 \tag{23}$$

$$\tag{24}$$

Sampling point 3

- There are currently 4 clusters

$$p(z_3 = 4 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00027 \quad (23)$$

$$p(z_3 = 5 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00020 \quad (24)$$

$$(25)$$

Sampling point 3

- There are currently 4 clusters

$$p(z_3 = \text{new} \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00000 \tag{23}$$

$$p(z_3 = 1 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 6.25 \\ 6.25 \end{matrix}, \mathbb{1} \right) = 0.48 \times 0.00000 \tag{24}$$

$$p(z_3 = 4 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00027 \tag{25}$$

$$p(z_3 = 5 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00020 \tag{26}$$

$$p(z_3 = 6 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{27}$$

Sampling point 3

- There are currently 4 clusters

$$p(z_3 = \text{new} \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00000 \tag{23}$$

$$p(z_3 = 1 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 6.25 \\ 6.25 \end{matrix}, \mathbb{1} \right) = 0.48 \times 0.00000 \tag{24}$$

$$p(z_3 = 4 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00027 \tag{25}$$

$$p(z_3 = 5 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00020 \tag{26}$$

$$p(z_3 = 6 \mid \vec{z_{-3}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \tag{27}$$

- After normalization:{new: 0.00 1: 0.00 4: 0.58 5: 0.42 6: 0.00}

Sampling point 3

- There are currently 4 clusters

$$p(z_3 = \text{new} \mid z_{-3}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00000 \quad (23)$$

$$p(z_3 = 1 \mid z_{-3}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 6.25 \\ 6.25 \end{matrix}, \mathbb{1} \right) = 0.48 \times 0.00000 \quad (24)$$

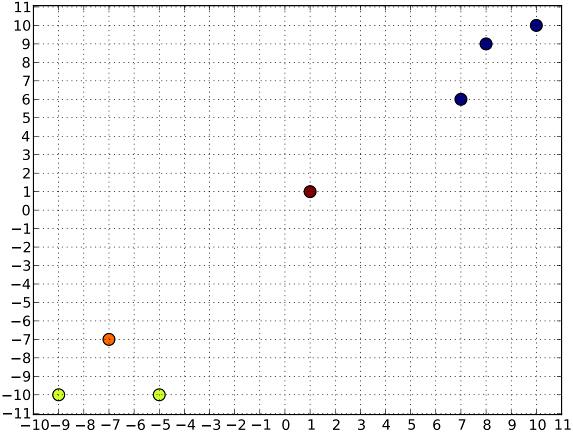
$$p(z_3 = 4 \mid z_{-3}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -2.50 \\ -5.00 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00027 \quad (25)$$

$$p(z_3 = 5 \mid z_{-3}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} -3.50 \\ -3.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00020 \quad (26)$$

$$p(z_3 = 6 \mid z_{-3}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{matrix} -9.00 \\ -10.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00000 \quad (27)$$

- After normalization: $\{$new: 0.00 1: 0.00 4: 0.58 5: 0.42 6: 0.00$\}$
- New assignment $= 4$

Assignments after sampling point 3



Sampling point 4

Compute the (proportional) probability of assigning data 4 to cluster 4 and 5.

Recall that $\alpha = 0.25$ and

$$p(x \,|\, \bar{x}) \propto \exp\left\{ -\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2} \right\} \quad (28)$$

i	x_1	x_2	z_i
0	10	10	1
1	8	9	1
2	7	6	1
3	-9	-10	4
4	-5	-7	
5	-7	-6	5
6	1	1	6

Sampling point 4

- There are currently 4 clusters

$$p(z_4 = 4 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00657 \quad (28)$$

$$(29)$$

Sampling point 4

- There are currently 4 clusters

$$p(z_4 = 4 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00657 \tag{28}$$

$$p(z_4 = 5 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -3.50 \\ -3.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00127 \tag{29}$$

(30)

Sampling point 4

- There are currently 4 clusters

$$p(z_4 = \text{new} \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00001 \tag{28}$$

$$p(z_4 = 1 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} 6.25 \\ 6.25 \end{array}, \mathbb{1} \right) = 0.48 \times 0.00000 \tag{29}$$

$$p(z_4 = 4 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00657 \tag{30}$$

$$p(z_4 = 5 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -3.50 \\ -3.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00127 \tag{31}$$

$$p(z_4 = 6 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} 0.50 \\ 0.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00001 \tag{32}$$

Sampling point 4

- There are currently 4 clusters

$$p(z_4 = \text{new} \mid z_{-4}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{array}{c}-5.00\\-10.00\end{array} \mid \begin{array}{c}0.00\\0.00\end{array}, \mathbb{1}\right) = 0.04 \times 0.00001 \tag{28}$$

$$p(z_4 = 1 \mid z_{-4}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}-5.00\\-10.00\end{array} \mid \begin{array}{c}6.25\\6.25\end{array}, \mathbb{1}\right) = 0.48 \times 0.00000 \tag{29}$$

$$p(z_4 = 4 \mid z_{-4}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}-5.00\\-10.00\end{array} \mid \begin{array}{c}-4.50\\-5.00\end{array}, \mathbb{1}\right) = 0.16 \times 0.00657 \tag{30}$$

$$p(z_4 = 5 \mid z_{-4}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}-5.00\\-10.00\end{array} \mid \begin{array}{c}-3.50\\-3.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00127 \tag{31}$$

$$p(z_4 = 6 \mid z_{-4}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}-5.00\\-10.00\end{array} \mid \begin{array}{c}0.50\\0.50\end{array}, \mathbb{1}\right) = 0.16 \times 0.00001 \tag{32}$$

- After normalization:{new: 0.00 1: 0.00 4: 0.84 5: 0.16 6: 0.00}

Sampling point 4

- There are currently 4 clusters

$$p(z_4 = \text{new} \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00001 \quad (28)$$

$$p(z_4 = 1 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} 6.25 \\ 6.25 \end{array}, \mathbb{1} \right) = 0.48 \times 0.00000 \quad (29)$$

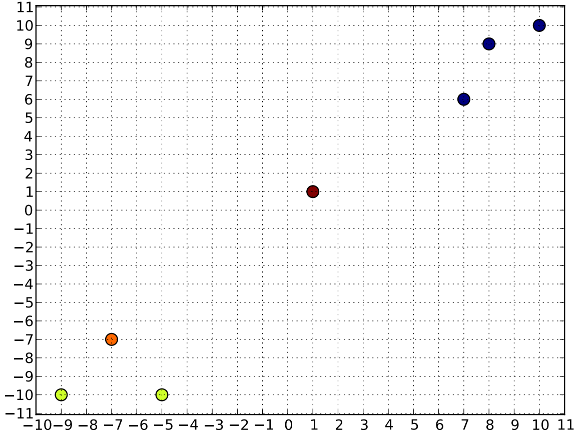
$$p(z_4 = 4 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -4.50 \\ -5.00 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00657 \quad (30)$$

$$p(z_4 = 5 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} -3.50 \\ -3.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00127 \quad (31)$$

$$p(z_4 = 6 \mid \vec{z_{-4}}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -5.00 \\ -10.00 \end{array} \mid \begin{array}{c} 0.50 \\ 0.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00001 \quad (32)$$

- After normalization:{new: 0.00 1: 0.00 4: 0.84 5: 0.16 6: 0.00}
- New assignment = 4

Assignments after sampling point 4



Sampling point 5

Compute the (proportional) probability of assigning data 5 to cluster 4 (but nothing else is viable).

Recall that $\alpha = 0.25$ and

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (33)$$

i	x_1	x_2	z_i
0	10	10	1
1	8	9	1
2	7	6	1
3	-9	-10	4
4	-5	-7	4
5	-7	-6	
6	1	1	6

Sampling point 5

- There are currently 3 clusters

$$p(z_5 = 4 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{2.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} -7.00 \\ -7.00 \end{array} \mid \begin{array}{c} -4.67 \\ -6.67 \end{array}, \mathbb{1} \right) = 0.32 \times 0.09470 \tag{33}$$

$$\tag{34}$$

Sampling point 5

- There are currently 3 clusters

$$p(z_5 = \text{new} \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00005 \quad (33)$$

$$p(z_5 = 1 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} 6.25 \\ 6.25 \end{matrix}, \mathbb{1} \right) = 0.48 \times 0.00000 \quad (34)$$

$$p(z_5 = 4 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{2.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} -4.67 \\ -6.67 \end{matrix}, \mathbb{1} \right) = 0.32 \times 0.09470 \quad (35)$$

$$p(z_5 = 6 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00002 \quad (36)$$

Sampling point 5

- There are currently 3 clusters

$$p(z_5 = \text{new} \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.00005 \quad (33)$$

$$p(z_5 = 1 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} 6.25 \\ 6.25 \end{matrix}, \mathbb{1} \right) = 0.48 \times 0.00000 \quad (34)$$

$$p(z_5 = 4 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{2.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} -4.67 \\ -6.67 \end{matrix}, \mathbb{1} \right) = 0.32 \times 0.09470 \quad (35)$$

$$p(z_5 = 6 \mid \vec{z_{-5}}, \vec{x}, \alpha) \propto \frac{1.00}{6 + 0.25} \mathcal{N}\left(\begin{matrix} -7.00 \\ -7.00 \end{matrix} \mid \begin{matrix} 0.50 \\ 0.50 \end{matrix}, \mathbb{1} \right) = 0.16 \times 0.00002 \quad (36)$$

- After normalization:{new: 0.00 1: 0.00 4: 1.00 6: 0.00}

Sampling point 5

- There are currently 3 clusters

$$p(z_5 = \text{new} \mid \vec{z}_{-5}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25} \mathcal{N}\left(\begin{array}{c} -7.00 \\ -7.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.00005 \tag{33}$$

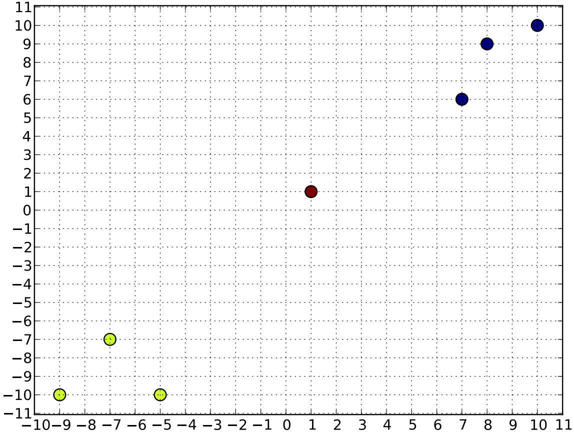
$$p(z_5 = 1 \mid \vec{z}_{-5}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -7.00 \\ -7.00 \end{array} \mid \begin{array}{c} 6.25 \\ 6.25 \end{array}, \mathbb{1} \right) = 0.48 \times 0.00000 \tag{34}$$

$$p(z_5 = 4 \mid \vec{z}_{-5}, \vec{x}, \alpha) \propto \frac{2.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -7.00 \\ -7.00 \end{array} \mid \begin{array}{c} -4.67 \\ -6.67 \end{array}, \mathbb{1} \right) = 0.32 \times 0.09470 \tag{35}$$

$$p(z_5 = 6 \mid \vec{z}_{-5}, \vec{x}, \alpha) \propto \frac{1.00}{6+0.25} \mathcal{N}\left(\begin{array}{c} -7.00 \\ -7.00 \end{array} \mid \begin{array}{c} 0.50 \\ 0.50 \end{array}, \mathbb{1} \right) = 0.16 \times 0.00002 \tag{36}$$

- After normalization:{new: 0.00 1: 0.00 4: 1.00 6: 0.00}
- New assignment = 4

Assignments after sampling point 5



Sampling point 6

Compute the (proportional) probability of assigning data 6 to a new cluster and cluster 1.
Recall that $\alpha = 0.25$ and

$$p(x \mid \bar{x}) \propto \exp\left\{-\sqrt{\left(x_1 - \frac{n}{n+1}\bar{x}_1\right)^2 + \left(x_2 - \frac{n}{n+1}\bar{x}_2\right)^2}\right\} \quad (37)$$

i	x_1	x_2	z_i
0	10	10	1
1	8	9	1
2	7	6	1
3	-9	-10	4
4	-5	-7	4
5	-7	-6	4
6	1	1	

Sampling point 6

- There are currently 2 clusters

$$p(z_6 = \text{new} \mid \vec{z_{-6}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{matrix} 1.00 \\ 1.00 \end{matrix} \mid \begin{matrix} 0.00 \\ 0.00 \end{matrix}, \mathbb{1} \right) = 0.04 \times 0.24312 \quad (37)$$

$$(38)$$

Sampling point 6

- There are currently 2 clusters

$$p(z_6 = \text{new} \mid z_{-6}^{\rightharpoonup}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.24312 \tag{37}$$

$$p(z_6 = 1 \mid z_{-6}^{\rightharpoonup}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \mid \begin{array}{c} 6.25 \\ 6.25 \end{array}, \mathbb{1} \right) = 0.48 \times 0.00060 \tag{38}$$

$$\tag{39}$$

Sampling point 6

- There are currently 2 clusters

$$p(z_6 = \text{new} \mid z_{\vec{-}6}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \middle| \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1} \right) = 0.04 \times 0.24312 \quad (37)$$

$$p(z_6 = 1 \mid z_{\vec{-}6}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \middle| \begin{array}{c} 6.25 \\ 6.25 \end{array}, \mathbb{1} \right) = 0.48 \times 0.00060 \quad (38)$$

$$p(z_6 = 4 \mid z_{\vec{-}6}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25} \mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \middle| \begin{array}{c} -5.25 \\ -6.75 \end{array}, \mathbb{1} \right) = 0.48 \times 0.00005 \quad (39)$$

Sampling point 6

- There are currently 2 clusters

$$p(z_6 = \text{new} \mid z_{-6}, \vec{x}, \alpha) \propto \frac{0.25}{6+0.25}\mathcal{N}\left(\begin{array}{c}1.00\\1.00\end{array} \mid \begin{array}{c}0.00\\0.00\end{array}, \mathbb{1}\right) = 0.04 \times 0.24312 \quad (37)$$

$$p(z_6 = 1 \mid z_{-6}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}1.00\\1.00\end{array} \mid \begin{array}{c}6.25\\6.25\end{array}, \mathbb{1}\right) = 0.48 \times 0.00060 \quad (38)$$

$$p(z_6 = 4 \mid z_{-6}, \vec{x}, \alpha) \propto \frac{3.00}{6+0.25}\mathcal{N}\left(\begin{array}{c}1.00\\1.00\end{array} \mid \begin{array}{c}-5.25\\-6.75\end{array}, \mathbb{1}\right) = 0.48 \times 0.00005 \quad (39)$$

- After normalization:{new: 0.97 1: 0.03 4: 0.00}

Sampling point 6

- There are currently 2 clusters

$$p(z_6 = \text{new} \mid \vec{z_{-6}}, \vec{x}, \alpha) \propto \frac{0.25}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \mid \begin{array}{c} 0.00 \\ 0.00 \end{array}, \mathbb{1}\right) = 0.04 \times 0.24312 \quad (37)$$

$$p(z_6 = 1 \mid \vec{z_{-6}}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \mid \begin{array}{c} 6.25 \\ 6.25 \end{array}, \mathbb{1}\right) = 0.48 \times 0.00060 \quad (38)$$

$$p(z_6 = 4 \mid \vec{z_{-6}}, \vec{x}, \alpha) \propto \frac{3.00}{6 + 0.25}\mathcal{N}\left(\begin{array}{c} 1.00 \\ 1.00 \end{array} \mid \begin{array}{c} -5.25 \\ -6.75 \end{array}, \mathbb{1}\right) = 0.48 \times 0.00005 \quad (39)$$

- After normalization:{new: 0.97 1: 0.03 4: 0.00}
- New assignment = 0

Assignments after sampling point 6