Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**

Clustering

Introduction to Data Science

Jordan Boyd-Graber and Michael Paul
SLIDES ADAPTED FROM LAUREN HANNAH

Slides adapted from Tom Mitchell, Eric Xing, and Lauren Hannah

Roadmap

- Classification: machines labeling data for us
- Previously: logistic regression
- This time: SVMs
  - (another) example of linear classifier
  - State-of-the-art classification
  - Good theoretical properties

Thinking Geometrically

- Suppose you have two classes: vacations and sports
- Suppose you have four documents

Sports

Doc_1: {ball, ball, ball, travel}
Doc_2: {ball, ball}

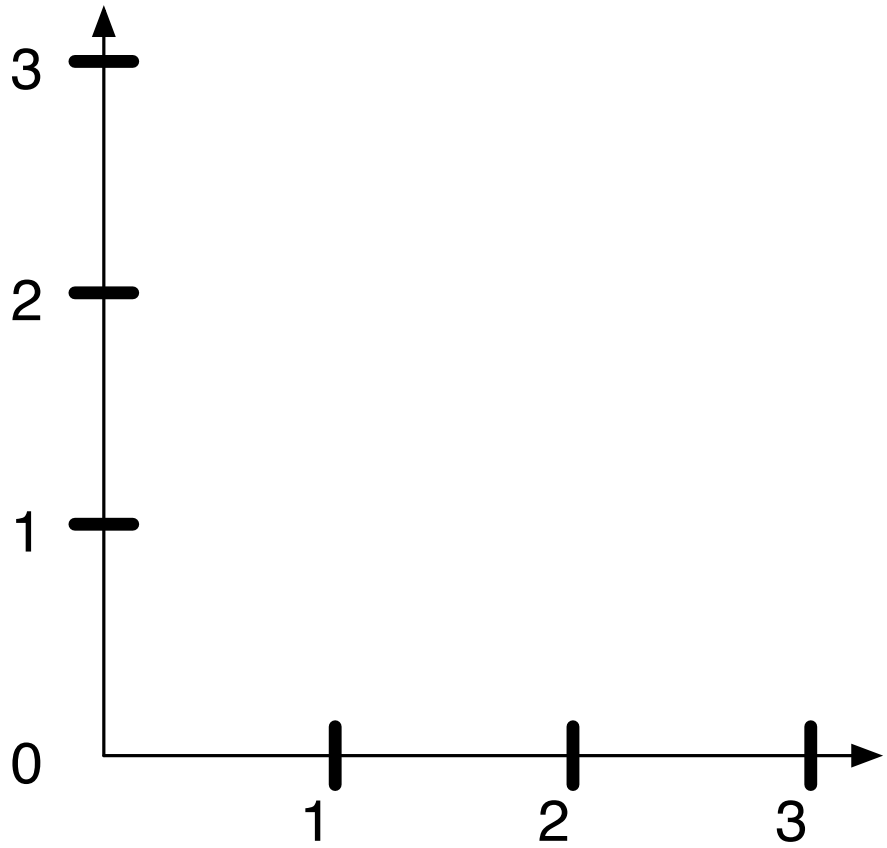
Vacations

Doc_3: {travel, ball, travel}
Doc_4: {travel}

- What does this look like in vector space?

Put the documents in vector space

Travel



Ball

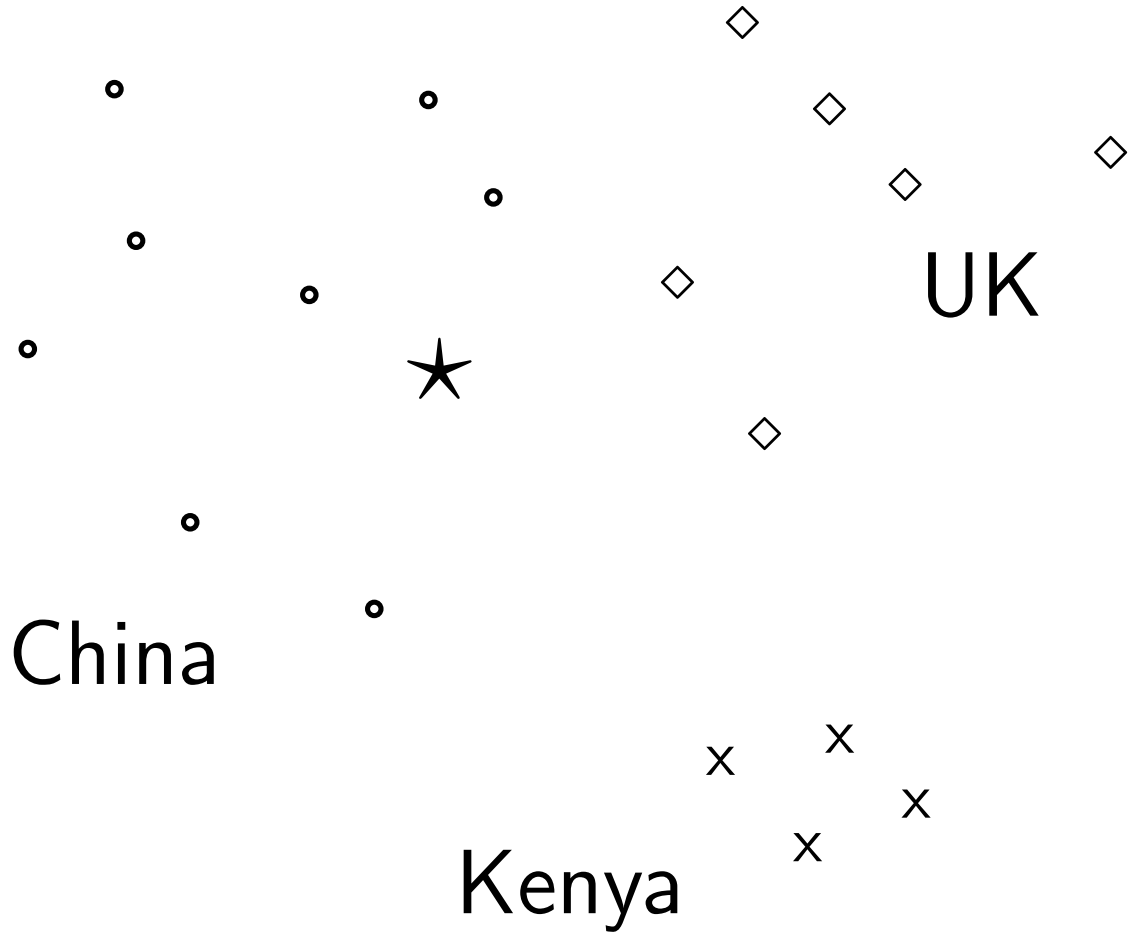
Vector space representation of documents

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 10,000s of dimensions and more
- How can we do classification in this space?

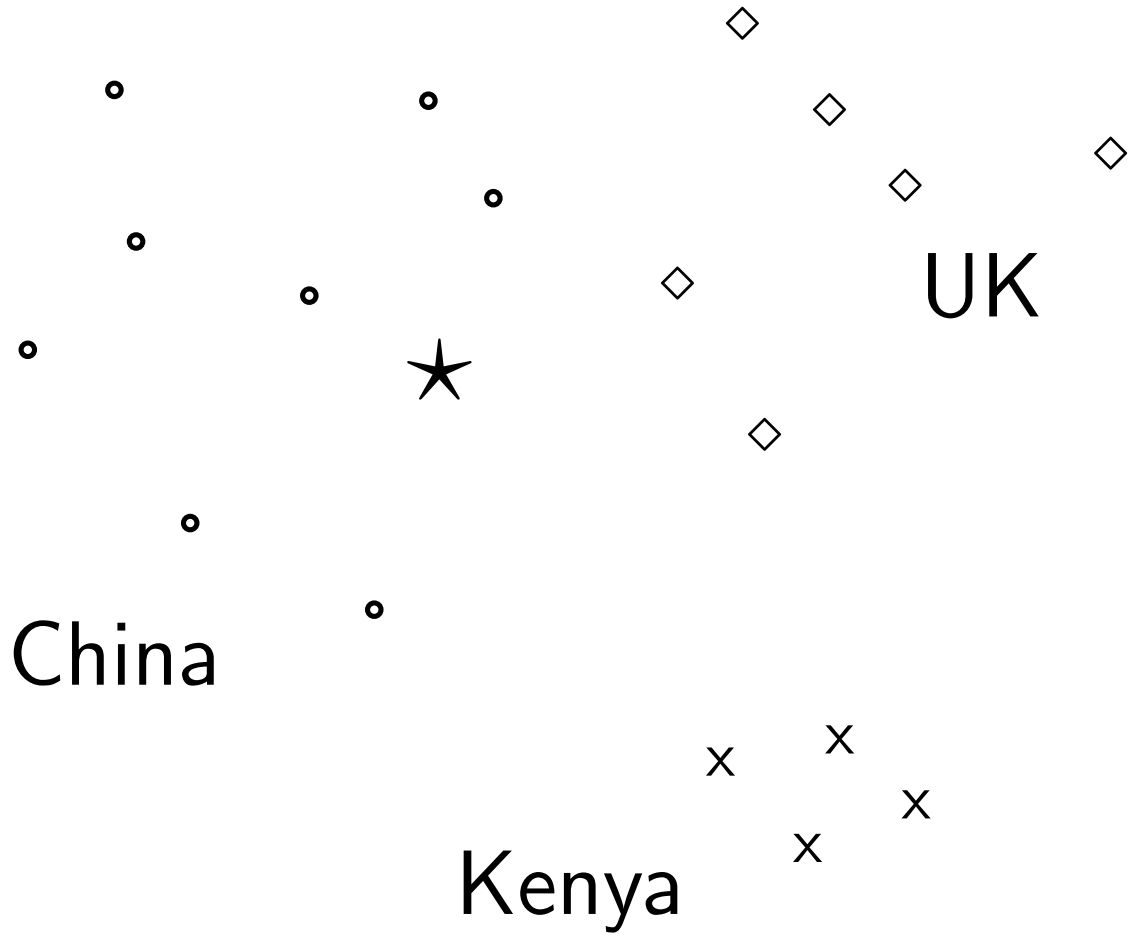
Vector space classification

- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a **contiguous region**.
- Premise 2: Documents from different classes **don't overlap**.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space

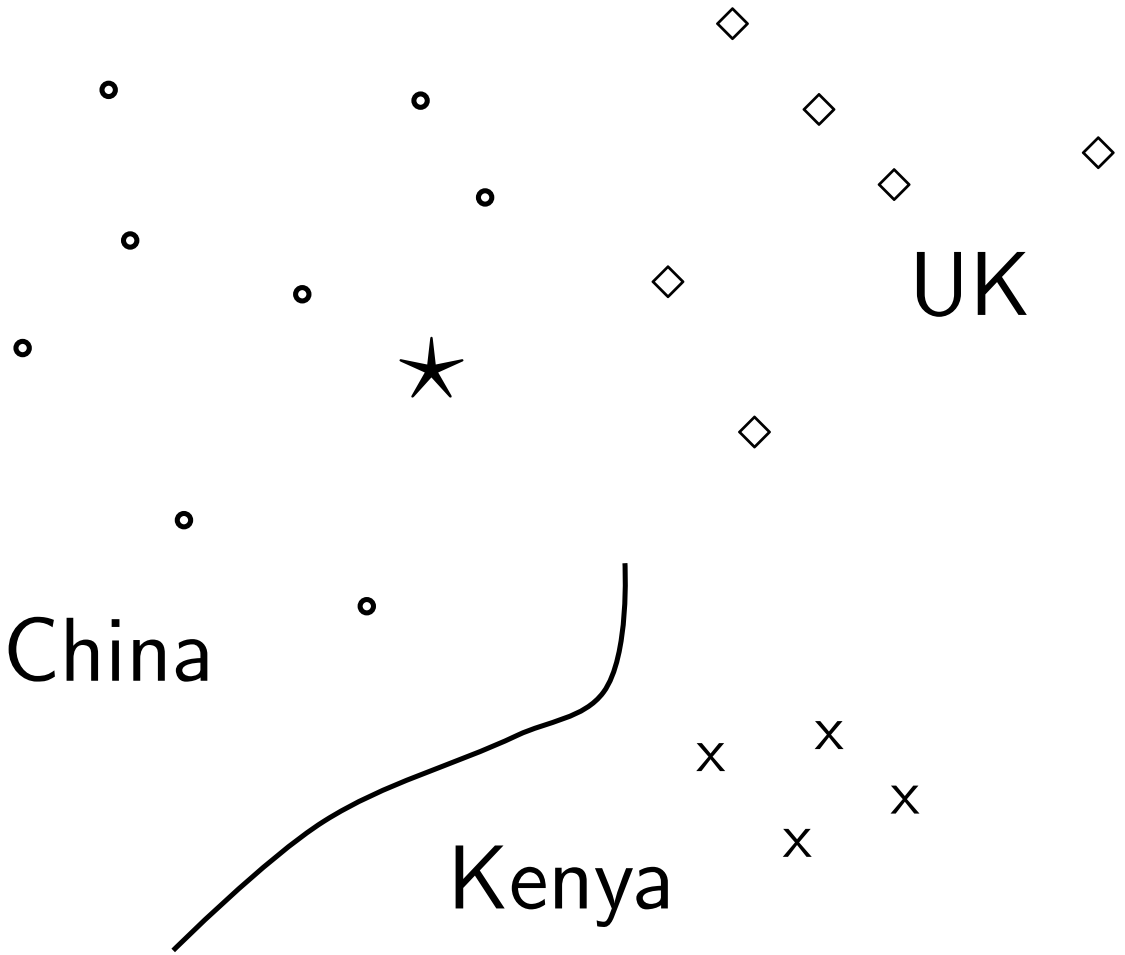


Classes in the vector space



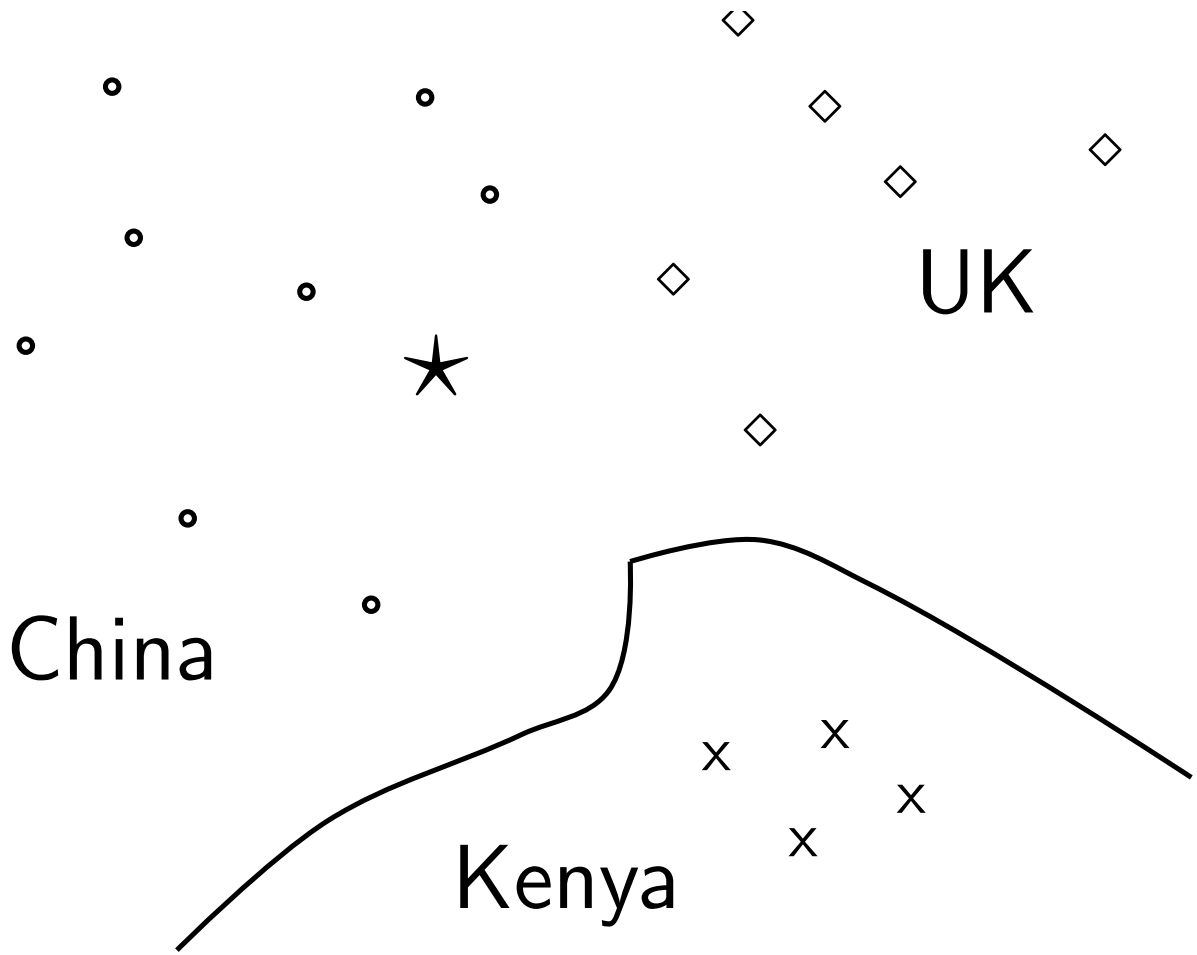
Should the document $\star$ be assigned to `China`, `UK` or `Kenya`?

Classes in the vector space



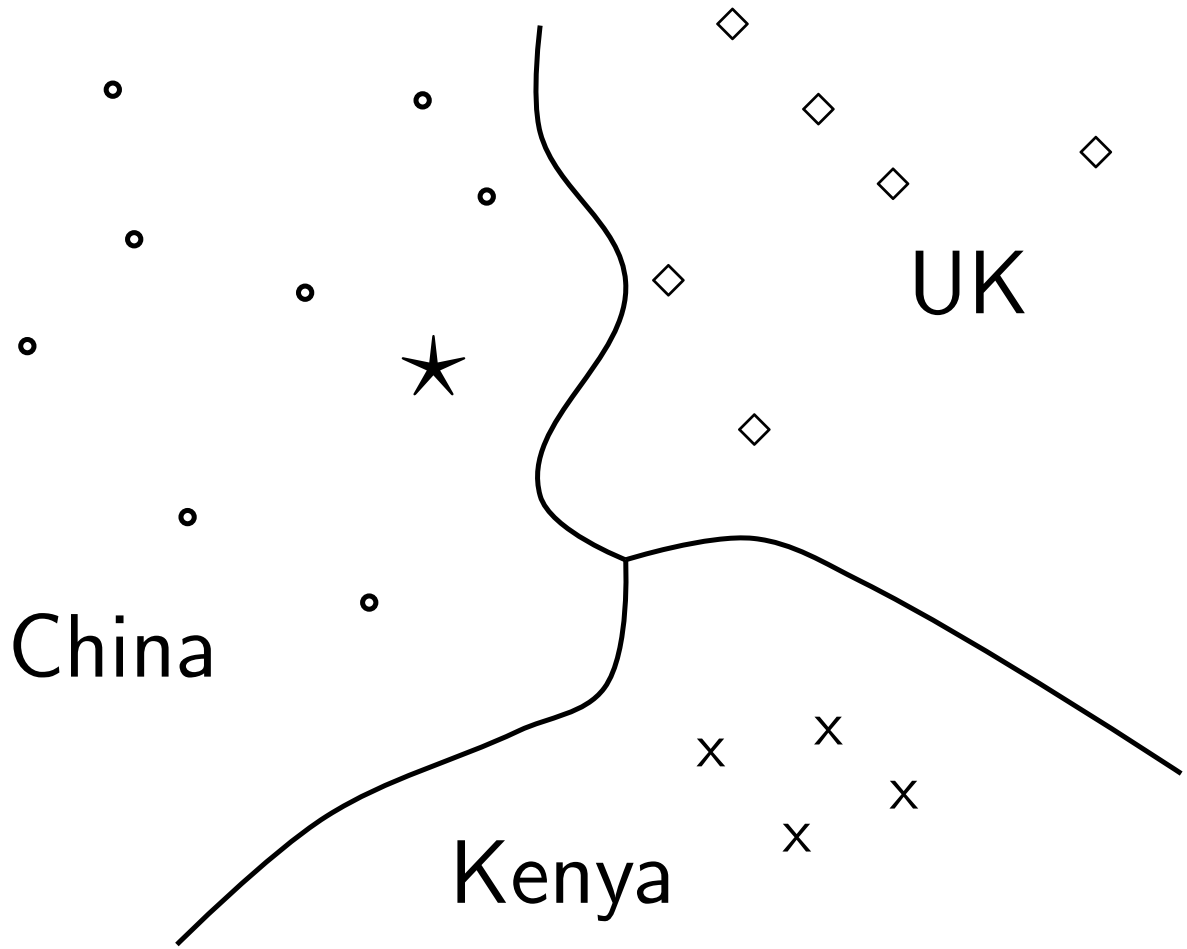
Find separators between the classes

Classes in the vector space



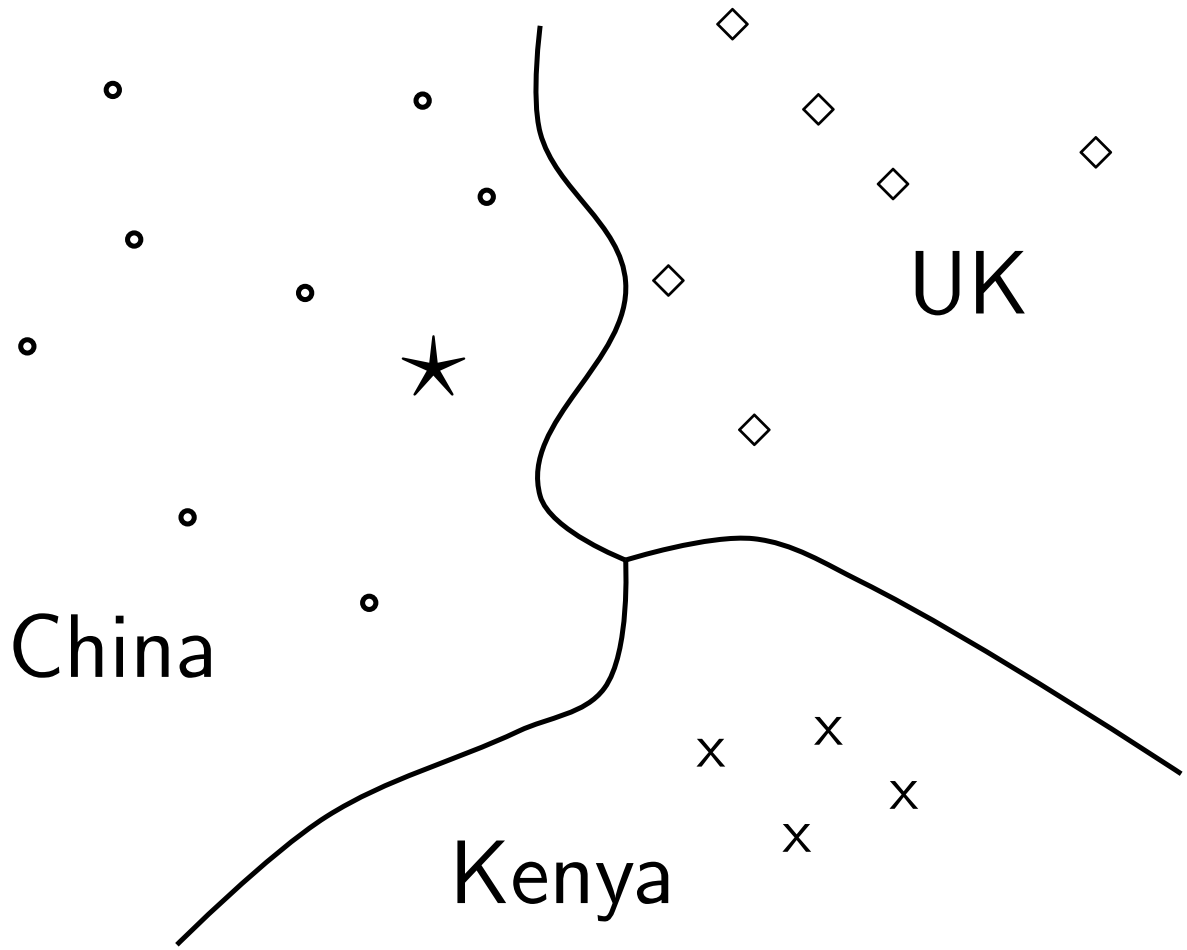
Find separators between the classes

Classes in the vector space



Based on these separators: $\star$ should be assigned to `China`

Classes in the vector space



How do we find separators that do a good job at classifying new documents like $\star$? – Main topic of today

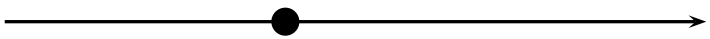
Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum $\sum_i \beta_i x_i$ of the feature values.
  - Classification decision: $\sum_i \beta_i x_i > \beta_0$? ($\beta_0$ is our bias)
  - . . . where β_0 (the threshold) is a parameter.
- We call this the **separator** or **decision boundary**.
- We find the separator based on training set.
- Methods for finding separator: logistic regression, linear SVM
- Assumption: The classes are **linearly separable**.

Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum $\sum_i \beta_i x_i$ of the feature values.
  - Classification decision: $\sum_i \beta_i x_i > \beta_0$? ($\beta_0$ is our bias)
  - . . . where β_0 (the threshold) is a parameter.
- We call this the **separator** or **decision boundary**.
- We find the separator based on training set.
- Methods for finding separator: logistic regression, linear SVM
- Assumption: The classes are **linearly separable**.
- Before, we just talked about equations. What's the geometric intuition?

A linear classifier in 1D



- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$

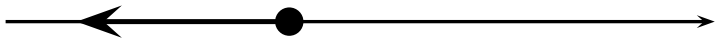
A linear classifier in 1D

- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$
- $x = \beta_0 / \beta_1$

A linear classifier in 1D

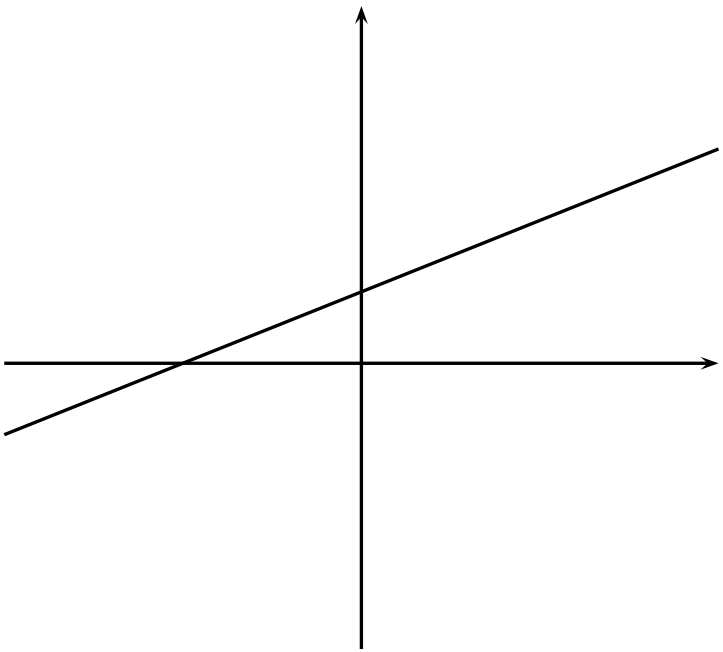
- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$
- $x = \beta_0 / \beta_1$
- Points (x_1) with $\beta_1 x_1 \geq \beta_0$ are in the class c.

A linear classifier in 1D



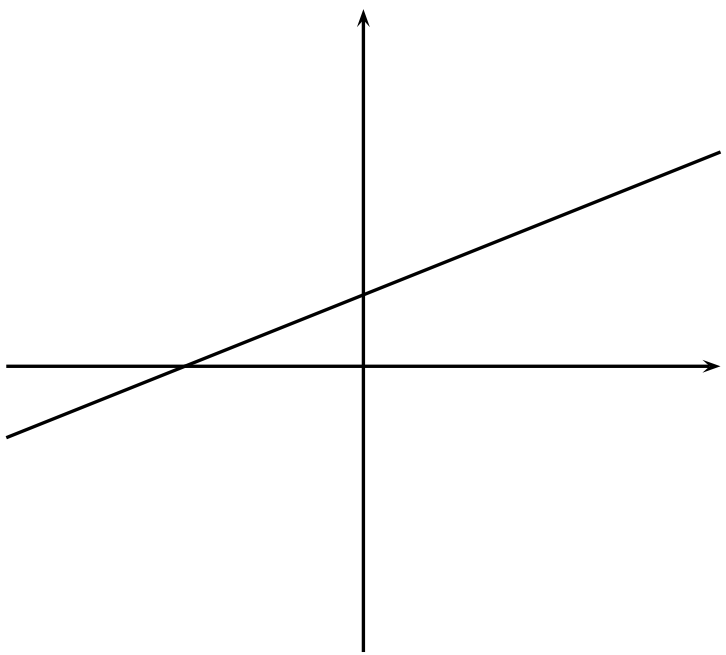
- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$
- $x = \beta_0 / \beta_1$
- Points (x_1) with $\beta_1 x_1 \geq \beta_0$ are in the class c.
- Points (x_1) with $\beta_1 x_1 < \beta_0$ are in the complement class $\overline{c}$.

A linear classifier in 2D



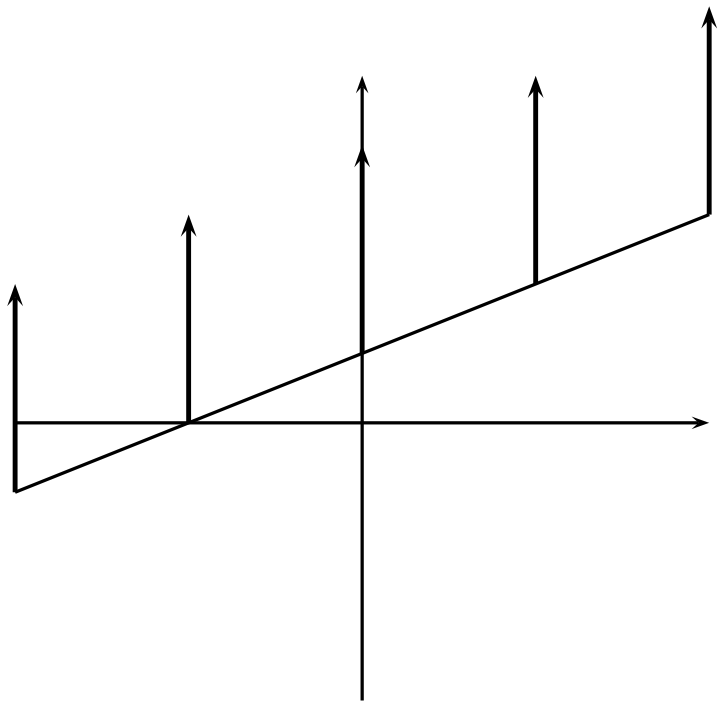
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$

A linear classifier in 2D



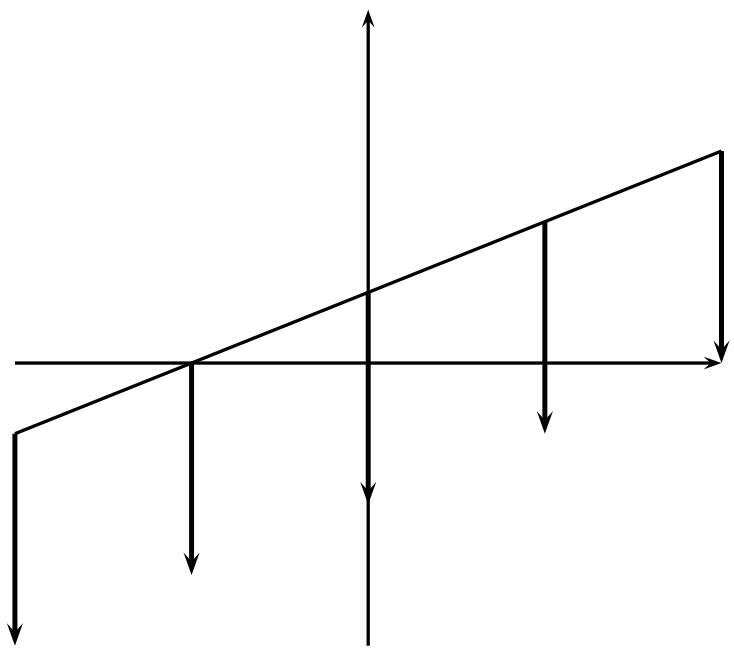
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$
- Example for a 2D linear classifier

A linear classifier in 2D



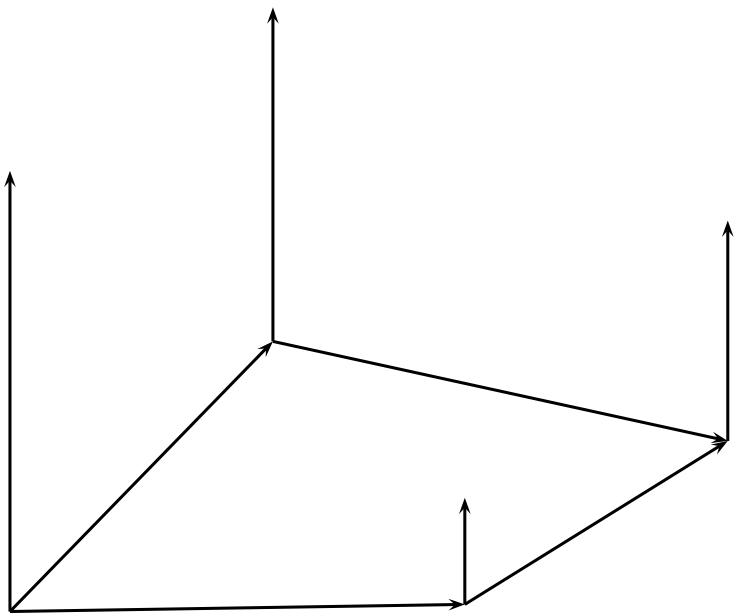
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$
- Example for a 2D linear classifier
- Points $(x_1\ x_2)$ with $\beta_1 x_1 + \beta_2 x_2 \geq \beta_0$ are in the class c.

A linear classifier in 2D



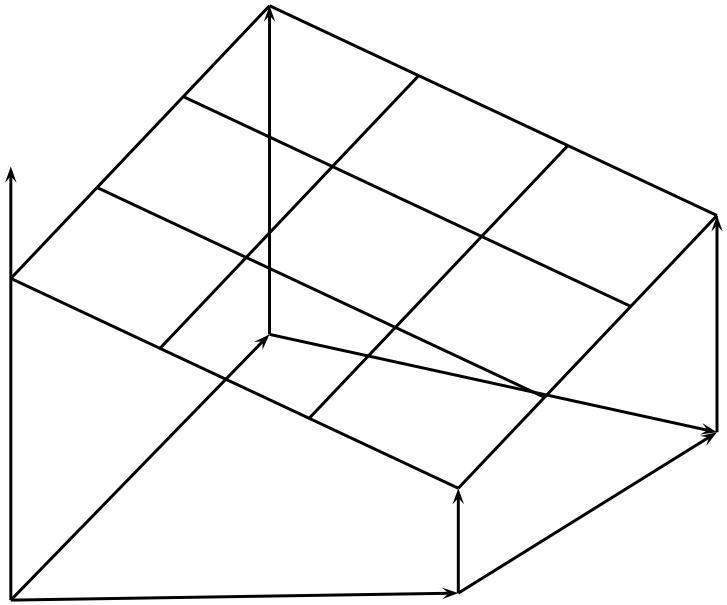
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$
- Example for a 2D linear classifier
- Points $(x_1\ x_2)$ with $\beta_1 x_1 + \beta_2 x_2 \geq \beta_0$ are in the class c.
- Points $(x_1\ x_2)$ with $\beta_1 x_1 + \beta_2 x_2 < \beta_0$ are in the complement class $\overline{c}$.

A linear classifier in 3D



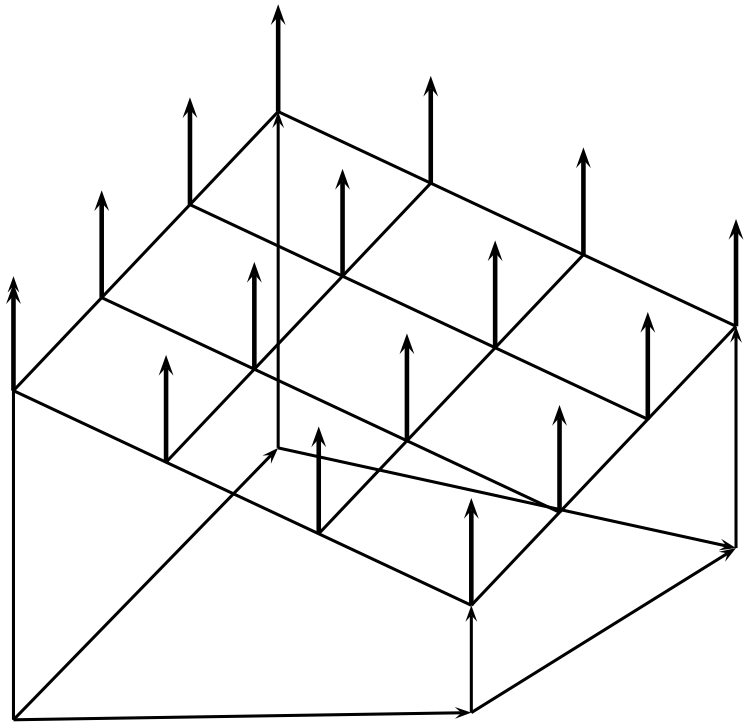
- A linear classifier in 3D is a plane described by the equation
$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$

A linear classifier in 3D



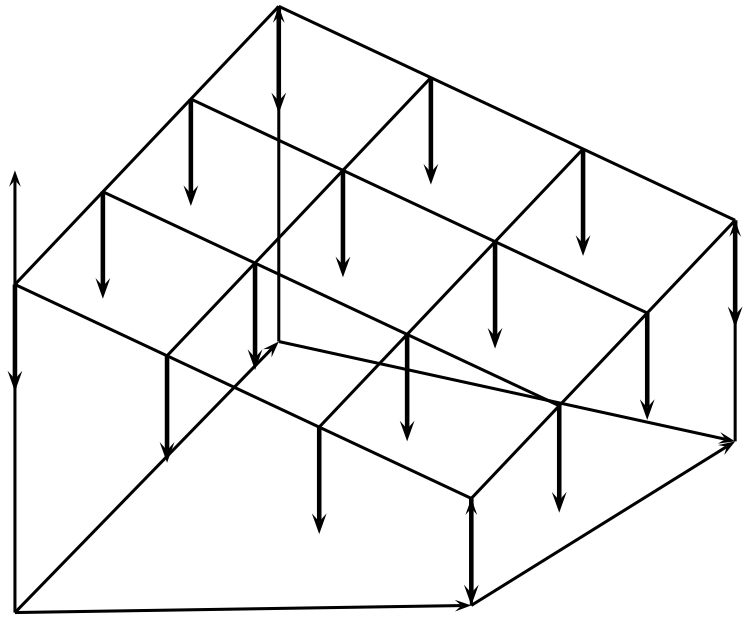
- A linear classifier in 3D is a plane described by the equation
  $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$
- Example for a 3D linear classifier

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$
- Example for a 3D linear classifier
- Points $(x_1\ x_2\ x_3)$ with $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \geq \beta_0$ are in the class c.

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$
- Example for a 3D linear classifier
- Points $(x_1\ x_2\ x_3)$ with $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \geq \beta_0$ are in the class c.
- Points $(x_1\ x_2\ x_3)$ with $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 < \beta_0$ are in the complement class $\overline{c}$.